ON THE CORRECTNESS OF VARIOUS APPROACHES IN THE EXTRACTION OF THE NUCLEUS PARAMETERS ON EXAMPLE OF ANALYSIS OF THR TWO-STEP γ-CASCADES IN ^{163}Dy COMPOUND NUCLEUS

V.A. Khitrov, A.M. Sukhovoj

Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Physics, 141980, Dubna, Russia

Pham Dinh Khang

National University of Hanoi

Vuong Huu Tan, Nguyen Xuan Hai

Vietnam Atomic Energy Commission

1 Introduction

All results of the analysis of experiment must be reliable, i.e., even having inevitable systematic errors, they must not lead to deliberately false conclusions about the nature of the studied phenomenon. For example, any selection and subsequent parameterization of nuclear models must provide the minimal probability of the appearance of wrong result, and must not lead to the generation of false hypotheses.

The greatest probability of obtaining the deliberately erroneous ideas about the studied process occurs at the stage of the extraction of its parameters from the measured data. First of all this is manifested in case of indirect experiments. The obvious example of this possibility is the extraction of information on the parameters of the compound-state cascade gamma-decay for nucleus with the high level density from the analysis of the two-step cascade intensities or any other measured spectra.

It is characteristic for these cases, that the dependence the intensity of the measured distribution on the level density ρ and the radiative strength functions k of the emission of nuclear reaction products is nonlinear. The corresponding systems of equations either are degenerated, or are close to the degenerated one's. And the existing ideas about the unknown parameters to greater or lesser extent are approximate or do not take into account the whole nuclear information accumulated by to now. The above mentioned ideas are well illustrated by the analysis of experimental data of the two-step cascade intensities occurring at thermal neutron capture in ^{162}Dy. Experience of study of two-step cascade intensities study by the Dubna group in more than 50 nuclei in the region $28 \leq A \leq 200$ (approximately a fourth of these data were obtained by V. Bondarenko group in Riga and J. Honzátko group in Rézh) allows us to assert that their analysis can either ensure obtaining or maximum reliable (at present) conclusions on the process of the cascade gamma-decay of nucleus with any highest density of levels [1,2], or lead to the completely erroneous conclusions on this studding process.

2 Specific character of obtaining experimental data for ρ and k in the nuclei with the high level densities

Experimental intensities of the two-step cascades between the neutron resonance and the group of low-lying levels from $E_f < 1$ MeV region are measured by Ge-detectors, i.e., the sequence of quanta in these cascades cannot be determined by apparatus. Accordingly, in any interval ΔE of cascade gamma-quanta energy E_γ the experimental intensity $I_{\gamma\gamma}^{exp} = I_{\gamma\gamma}(E_1)+I_{\gamma\gamma}(E_2)$ is the sum of unknown terms, corresponding either to the primary, or to the secondary cascade transition. Their energies E_1 or E_2 in this case satisfy the obvious conditions: $E_\gamma \leq E_1 \leq E_\gamma + \Delta E$ and $E_\gamma \leq E_2 \leq E_\gamma + \Delta E$. Taking into account the physical limitation, it is only possible to conclude, that the cascade intensity with the fixed order of the cascade quanta always lies in the interval of its value from zero to $I_{\gamma\gamma}^{exp}$. And this circumstance leads to a catastrophic increase [3] of the interval widths both values ρ and k, which can reproduce $I_{\gamma\gamma}^{exp}$ for all possible values of energies of the cascade gamma-transitions $E_1 + E_2 = B_n - E_f$ with one and the same χ^2. Moreover - not only for the cascades for one fixed low-lying level, but also for their set for several final levels together.

This problem can be solved only by determining the cascade intensities in the function of their primary gamma-transition energy E_1:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_\lambda}\frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f}\frac{\Gamma_{\lambda i}}{<\Gamma_{\lambda i}> m_{\lambda i}}n_{\lambda i}\frac{\Gamma_{if}}{<\Gamma_{if}> m_{if}}. \quad (1)$$

The functional (1) depends both on the ratios of partial and total radiative widths Γi of primary E_1 and secondary E_2 gamma-transitions in the cascades between the levels λ, i and f, and on the number $n(m) = \rho \times \Delta E$ of levels excited in this case in different energy intervals.

The practical possibility of determining this intensity was proposed by the authors [4]. The most probable error in the operation of division the experimental spectrum into the parts $I_{\gamma\gamma}(E_1)$ and $I_{\gamma\gamma}(E_2)$ with good precision can be determined from the approximation [5] in the assumption, that the distributions of random values of the cascade intensities with the dipole electric and magnet primary cascade transitions are described by a pair of distributions with some averages and by dispersion slightly exceeding the dispersion of the Porter-Thomas distribution.

The degree of detailing, with which can be extracted from observed $I_{\gamma\gamma}(E_1)$ the form of energy dependence of both level density, and radiative strength functions is determined by the optimum width of averaging interval ΔE of the cascade intensities. A good compromise between the real possibilities of contemporary detector and computer technology and the inevitable fluctuations of partial widths is reached at the width of interval $\Delta E = 50$ - 100 keV or larger, into which the excitation energy of nucleus is divided . Type of successive transition (dipole electrical or magnetic one), spin and parity of the intermediate level i excited in this case are uniquely determined by known values J^π levels of λ and f. A very small intensity of the cascades between the levels with spin difference $|J_\lambda - J_f| > 2$ exclude the need for to use the transitions of the highest multipolarities in the analysis [1,2]. A possibility of their including in the analysis does not cause doubts, howerer, practical expediency is completely absent.

In the cascades to the levels with the spins $|J_\lambda - J_f| \leq 2$ the admixture of E2-transitions is taken into account [1,2] automatically due to the appropriate distortion of the radiative

strength functions of pure M1-transitions. The direct calculation of gamma-transitions of the highest multipolarities is required in this case only to reproduce the intensity of cascades to the level $E_f = 73$ keV with the spin $J = 7/2$, the intensity of which is less than errors of the remaining values of $I_{\gamma\gamma}$. And the including of the additional free parameter in the analysis with conservation of the number of the experimental points will unavoidably increase the uncertainty of the obtained values ρ and k.

For convenience of the direct comparison the obtained radiative strength functions and E1-, and M1-transitions in the nuclei of different mass A it is expedient to determine in the form:

$$k = \Gamma_{\lambda i}/(E_\gamma^3 \times A^{2/3} \times D_\lambda) \tag{2}$$

and to use the experimentally measured and approximated into [6] near B_n ratios $k(M1)/k(E1)$ for mutual standardization in the iterative processes [1,2].

Calculation of the internal conversion of gamma-quanta and form of Ge-detector line at the registration threshold $E_\gamma > 520$ keV is not required. The intensity of cascades in the expression (1) is proportional to the derivative $dk/ > dE$ and, in the first approximation, is inversely proportional to ρ. This ensures the maximum sensitivity of the experiment in the region of the smallest values of ρ. I.e., in the excitation energy regions, where the influence of nuclear structure on the parameters of the investigated nuclear reaction must be a maximum. A fundamentally different form of the relation between the unknown parameters in comparison with the usual evaporative ones and the gamma-spectra provides the smallest influence of correlation of the parameters on their real error.

Interval of the intermediate level density values and partial radiative widths of cascade gamma-transitions, reproducing an experiment with the assigned accuracy, can be determined from (1) with the smallest possible systematic error only with using [2] the additional experimental information about the relation of the partial widths of the primary and secondary transitions of one and the same multipolarity and energy; values of the total radiative width of compound-state [7] and the information about the density of low- [8] and high-lying [7] levels (here - neutron resonances).

The values ρ and k obtained from the system of equations (1) are very rigidly determined (interval of their possible values is always small) and have principally unremoval deviations with the results obtained within the framework of other known procedures [9,10] and the conventional ideas [11-14] on this nuclear parameters.

3 Specific character of the existing model notions for the nuclei with high level density

Actually, the only basis of the existing model notations about ρ used in practice by experimenters is the notion of nucleus as a system of noninteracting Fermi gas [11] and the conclusion following from it about a smooth change in the level density with an increase in the excitation energy of nucleus. Like the Gilbert-Cameron model [12] analogous in essence, it completely ignores the existence of the excitations of vibration (and rotation - for the deformed ones) types in the nucleus and their interaction with quasi-particles. Therefore the indicated model notions cannot but give a significant error in the predicted values of ρ.

Unfortunately, this situation was not changed much by present-day models of the level density. For example, the general model of superfluid nucleus [13] directly takes into account the mutual transformation of nucleus from the superfluid state into the usual ones. However, its parameters are selected (and fixed into the model) on the basis of experimental information about ρ, containing unknown but significant systematic errors. First of all, it may say on the data about ρ, which obtained from the evaporated nucleons spectra. Usually authors of the corresponding experiments carry out a subjective selection of the potentials used in the optical nuclear model to calculate the probability of emission of the reaction product by an excited nucleus. Corresponding experimental data are quite absent up to now and resulted systematic error cannot be determine even in principle.

Criteria of the corresponding selection are not published, therefore it is only possible to assume, that it is just a degree of "smoothness" of the obtained dependence $\rho = f(E_{ex})$. Similar selection of versions of analysis is also inevitable in the procedure of extraction of ρ and k from the spectra of primary gamma-transitions during decaying of the levels with different energy of their excitation in the reactions of the type of (d,p) or $(^3\mathrm{He},\alpha,\gamma)$. Its need is caused both by the enormously high coefficients of the spectrum errors transfer to the errors of the determined parameters ρ and k, and by other specific sources of significant systematic errors.

Parametrization [16] of model of the partial level density for n-quasiparticles [17] showed, that the contemporary model notions about the nucleus are quite capable to reproduce "non-smooth" functions $\rho = f(E_{ex})$ with high accuracy. It is also not contradict to the known physical picture of interactions and the mutual transformation of boson and fermionic macrosystems into each other.

4 The two-step cascade experimental data analysis in ^{163}Dy

Experimental data on the intensities of the two-step cascades from the $^{162}\mathrm{Dy}(n,\gamma)$ reaction [18] have been obtained by us with substantially smaller statistics, than in [19] and with a slightly different positions of sample and Ge-crystals. These circumstances can distort in different ways, first of all, the value of the absolute intensity of cascades. Comparison of the absolute values of their sum from the interval $(E_1 + E_2)/2 \pm 1$ MeV (% to the decay) for the data [18,19] is presented in the table. The maximum value of E_f in it corresponds to the data [18], and is 2 times less than in [19] because of the substantial difference of the efficiencies of the detectors used and intensity of thermal neutron beams.

Table.

$E_1 + E_2$, keV	E_f, keV	$I_{\gamma\gamma}$ [18]	$I_{\gamma\gamma}$ [19]
6272	0	3.7	3.3
6198	73	0.4	0.2
6020	251	2.1	2.2
5920	351	3.8	3.3
5881	389	3.1	2.9
5847	421+427	3.9	absent

As seen from these data, the observed difference between the experimental data cannot substantially [20] influence on the values ρ and k, which reproduce precisely [21] the intensities of cascades. Consequently, the fundamental incompatibility of conclusions obtained from the different variants [18,19] of analysis indicates the incorrectness, at least, of one of them.

This situation is clearly illustrated in Figs. 1-3 by results of the search for values of ρ and k, maximally accurately reproducing experimental intensities of two-step cascades in 5 spectra measured by us (cascades of $E_1 + E_2 = 6198$ keV are not included in the analysis in view of circumstances mentioned above).

It follows from these data, that the experimental spectra of ^{163}Dy with the unknown sequence of quanta in the cascades can be reproduced with the maximally high accuracy by an infinite number of different functional dependencies of ρ and k. The ratio of the maximal and of the minimal possible values of these parameters reaches ~ 100. The reason for this spread is obvious: for example, very great significances of k in the region of energy $E_1 \sim 1$ MeV appear because the corresponding experimental intensities are approximated in a few cases as corresponding only (or in general only) to primary gamma-transitions. Naturally, that to define one of the terms from the known value of their sum the additional experimental data is necessary!) And with such enormous spread of the values of the parameters one should take into consideration, that the interval of possible values of ρ and k in Figs. 2,3 (because of the small number of realized random processes [1]) is determined very approximately. But it cannot be less, than that follows from the data on Figs. 2,3.

Approximation of the of ρ obtained in [21] (with the aid of the procedure [1] from the values $I_{\gamma\gamma}(E_1)$ only) by partial densities of 1-, 3-, 5-, 7- quasi-particle excitations [16] is represented in Fig. 4 for the version of the logarithmic dependence of correlation functions on the excitation energy of nucleus. The comparison of the sums of radiative strength functions with the relative percentages of n-quasiparticle levels is executed in Fig. 5. The coefficient of a vibration increase of the level density in the case of ^{163}Dy is very close to other nuclei data and equal to 13. This means, that more than 90% of levels lower than the threshold of the appearance of 5-quasiparticle excitations has wave functions with significant components of vibration type. However the concrete type of phonons, cannot be determined only from the intensities of cascades without the attraction of additional information.

5 Conclusion

The model-free analysis of experimental data on the intensities of two-step cascades in ^{163}Dy shows, that the basic special features of the cascade gamma-decay of its compound-state completely correspond to those observed in other nuclei. And they do not require the introduction of some additional hypotheses. Moreover the reliable picture of the studied process can be obtained only from the distributions of the intensities of cascades for the determined values of their primary gamma-transition energies .

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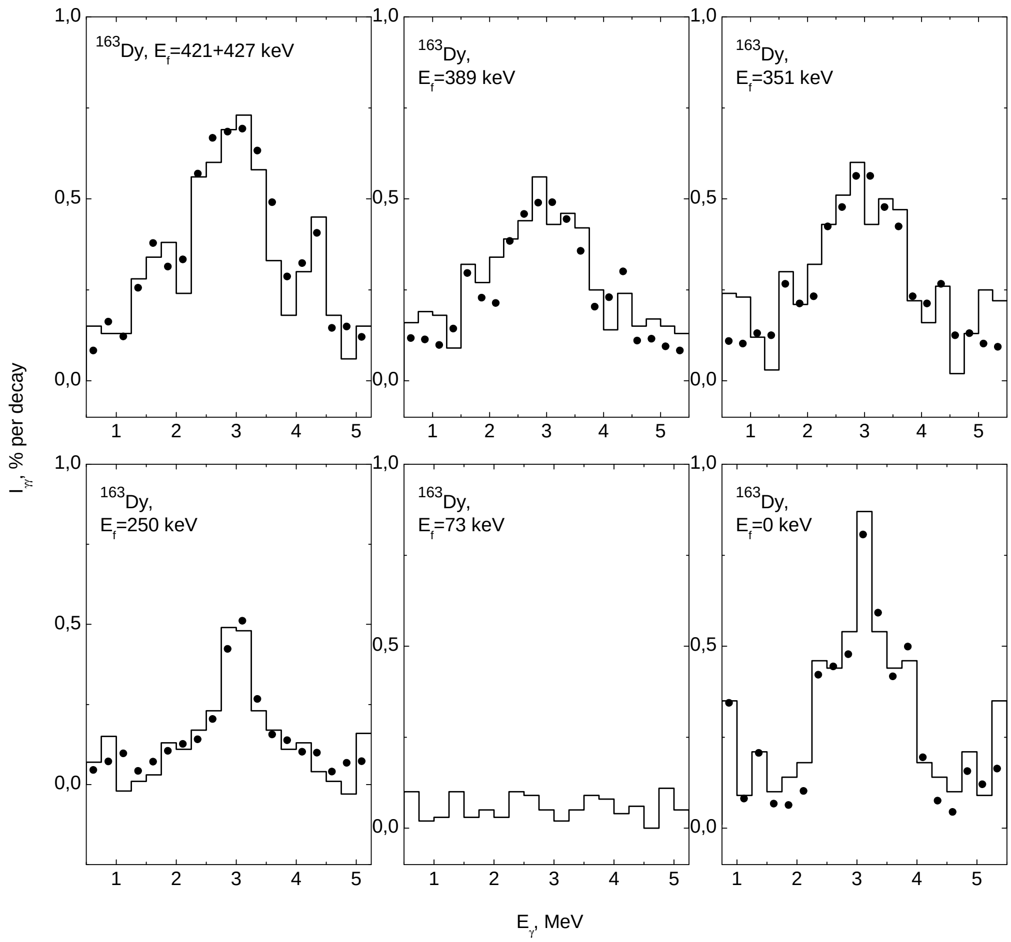

Fig. 1 Lines are the total experimental intensities (in % per decay) of two-step cascades (summed in energy bins of 250 keV) as a function of any cascade quanta energy (is multiplied by 2). Points - typical approximation for any pair of the level density and radiative strength functions from those given in Figs. 2,3.

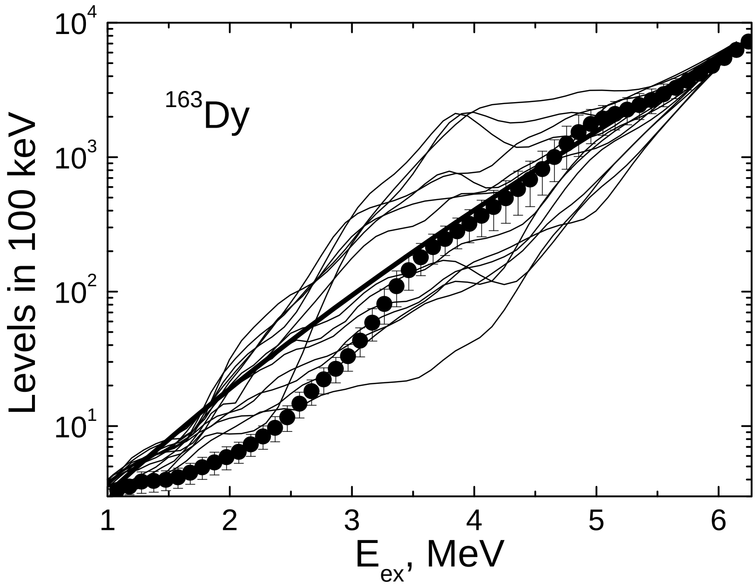

Fig. 2 Points with error bars are the interval of probable values of the level density enabling the reproduction of the experimental cascade intensity (as a function of primary transition energy) and total radiative width of capture state. The thick line represents predictions of the model [11]. Thin lines - versions of the obtained random functions reproducing the intensities from Fig. 1.

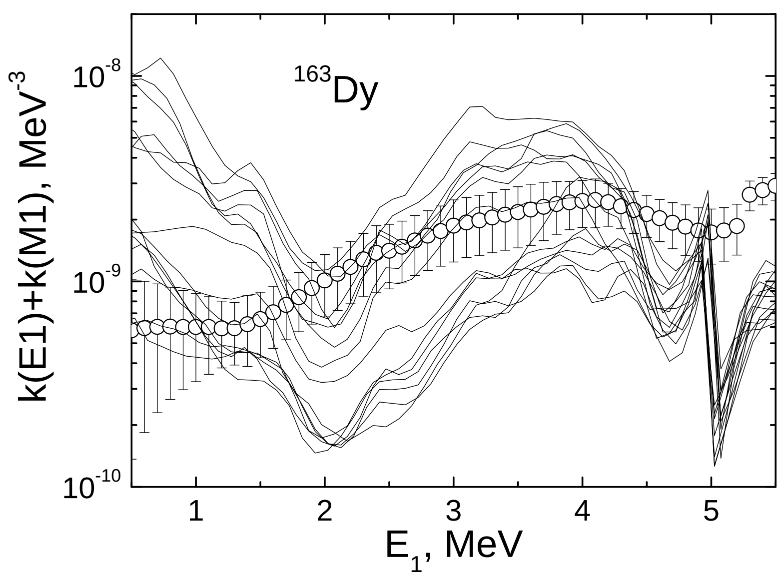

Fig. 3 The probable interval of the sum radiative strength function $k(E1) + k(M1)$ (points with error bars) providing the reproduction of the experimental data. Thin lines - versions of the obtained random functions reproducing the intensities from Fig. 1.

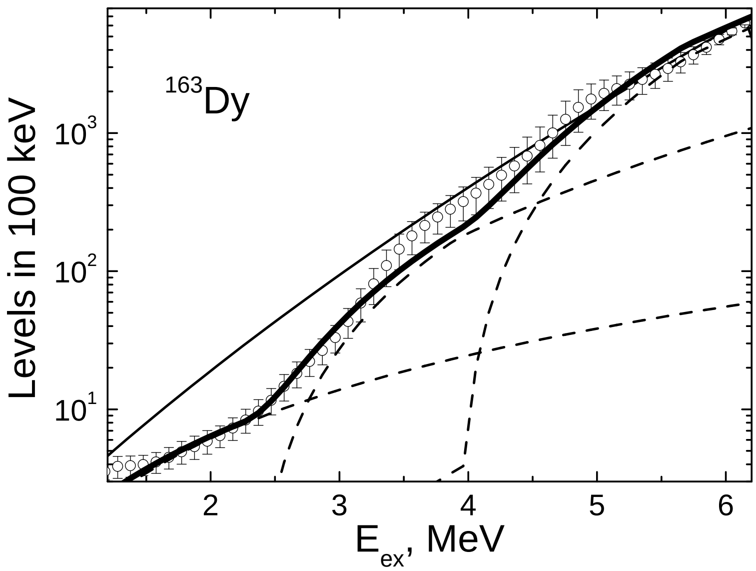

Fig. 4 Dotted line - approximation of the most probable ^{163}Dy level density by the partial densities of 3-, 5- and 7- quasi-particle excitations with the most probable coefficient of its collective enhancement factor. Thick line - sum of partial densities. Thin line is prediction of the model [11].

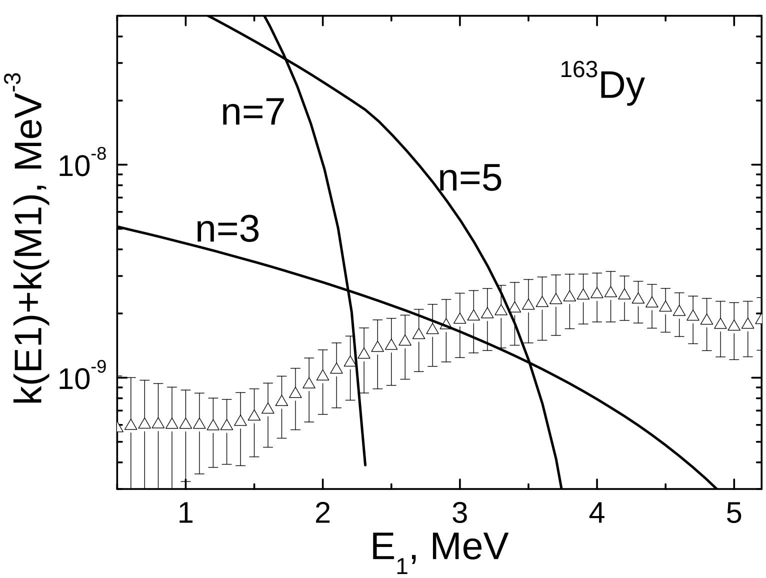

Fig. 5 Points with the errors - most probable sum of radiative strength functions. Dotted line - relative percentage of levels, corresponding to excitation n-quasiparticles for the energy of their excitation $E_i = B_n - E_1$.